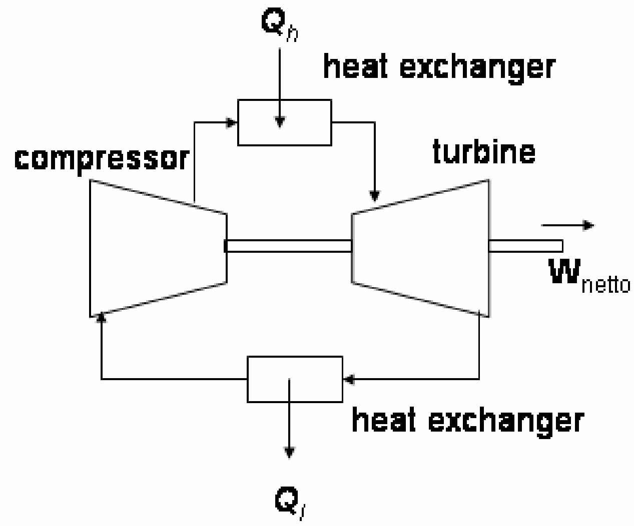
Intermediate Exam

Master Course Energy Conversion and Environmental Technologies
Date: 16 December 2005

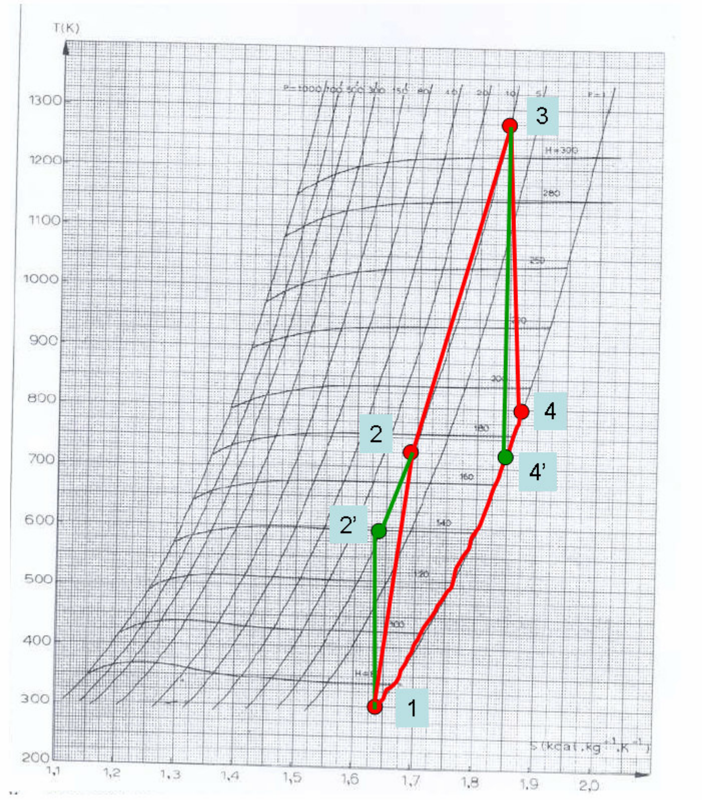
Answers:

1. Efficiency of a gas turbine

a. [10%]



b. [10%] 1-2-3-4; **d**. [10%] 1-2'-3-4'



c. [20%]

The thermal efficiency of this air-standard Brayton cycle can be calculated with

$$\eta = \frac{(h_3 - h_4) - (h_2 - h_1)}{(h_3 - h_2)}$$

	P (bar)	T (K)	H (kcal/kg)	
State 1	1	300	72	$\eta = \frac{(130)-(100)}{(148)} = 20.3\%$
State 2	10	725	172	
State 3	10	1275	320	
State 4	1	800	190	

e. [20%]

Use $\eta = \frac{(h_3 - h_4) - (h_2 - h_1)}{(h_3 - h_2)}$ with states 1-2'-3-4'

	P (bar)	T (K)	H (kcal/kg)	
State 1	1	300	70	$\eta = \frac{(150) - (70)}{(180)} = 44.4\%$
State 2'	10	600	140	
State 3	10	1275	320	
State 4'	1	800	170	

f. [10%] Isentropic efficiencies:

Turbine: $\eta_{turb} = \frac{(h_3 - h_4)}{(h_3 - h_{4'})} = \frac{(320 - 190)}{(320 - 170)} = 86.7\%$

Compressor: $\eta_{comp} = \frac{(h_{2'} - h_1)}{(h_2 - h_1)} = \frac{(140 - 70)}{(172 - 70)} = 68.8\%$

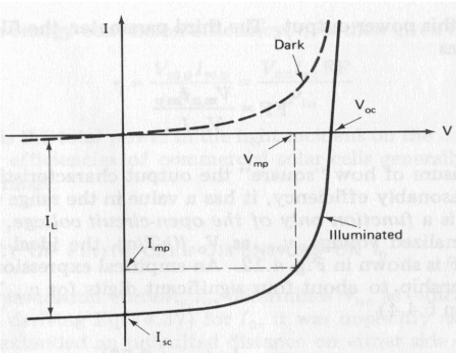
g. [10%] Yes, compression ratio increase leads to efficiency increase. Larger area in T-S diagramme.
Or from calculation example for isentropic case, *p*=20 bar, which changes state 2', 3' and 4. Turbine inlet temperature (T3) left constant.

	P (bar)	T (K)	H (kcal/kg)	
State 1	1	300	70	$\eta = \frac{(180) - (95)}{(155)} = 54.8\%$
State 2'	20	700	165	
State 3'	20	1275	320	
State 4	1	600	140	

h. Increasing pressure leads to higher efficiency, but may lead to large systems. Optimization for efficiency is replaced by optimization for work per mass flow, when system size is setting constraints (see Moran, page 447).

2. Photovoltaic conversion

a. [20%] Efficiency $\equiv (V_{oc} \times J_{sc} \times FF) / P_{in}$



b. [20%]

$$0 = J_0 \left[\exp\left(\frac{V_{oc}}{V_{th}} \right) - 1 \right] - J_{sc} \Rightarrow \frac{V_{oc}}{V_{th}} = \ln \left[\frac{J_L}{J_0} + 1 \right] \Rightarrow V_{oc} \approx V_{th} \ln \left[\frac{J_L}{J_0} \right]$$

c. [20%]
At 1 sun: $0.620 \approx V_{th} \times \ln (J_L/J_0) = 26 \text{ mV} \times C$
At X Suns: V_{oc} (X suns) $= 26 \times C' = 26 \times (C + \ln X) = V_{oc} + 26 \ln X$

Calculate logarithms: ln 10 = 2.30; ln 100 = 4.61; ln 1000 = 6.91

V_{oc} (10 suns) = 620 + 60 = 680 mV
V_{oc} (100 suns) = 620 + 119 = 739 mV
V_{oc} (1000 suns) = 620 + 179 = 799 mV

d. [20%]
$0.18 = (V_{oc} \times J_{sc} \times FF) / P_{in} = 0.62 \times J_{sc} \times FF$

Choose FF = 0.8 or something in that range: J_{sc} = 36.29 mA/cm^2
J_{mp} is than typically 0.9×36.29 = 32.66 mA/cm^2
(instead of 0.9, any choice between 0.8 and <1.0 would be okay)

e. [20%]

$$J(V) = J_0 \left[\exp\left(\frac{qV}{nkT} \right) - 1 \right] - J_L = J_0 \left[\exp\left(\frac{V}{nV_{th}} \right) - 1 \right] - J_L$$

For fixed values of J_{sc} and V_{oc}, the FF and thus the efficiency will always be lower for n >1. This is because for a given voltage V ($0<V<V_{oc}$) the diode with n>1 conducts more current than the one with n=1 (without illumination). *This means that for n=2 the curve is less "square"*. For instance, at V=500 mV one can calculate the ratio of currents for n=2:
→ $J(n=1)/J(n=2) \approx (J_{0,1}/J_{0,2}) \times \exp(qV/2kT) \approx 0.1$

For $V = V_{oc}$ the ratio = 1, while for $V > V_{oc}$, the ratio >1.
Note that one can easily calculate (using values for V_{oc} and $J_{sc} = J_L$): $J_{0,1} = 1.58\ 10^{-12}$ A/cm^2, $J_{0,2} = 2.4\ 10^{-7}$ A/cm^2, and exp(…) = 1.48 10^4. These values hold for the values estimated in [d].

3. Nuclear fission

a. [30%]

$$E_B = \left(Zm_H + Nm_n - {}^{A}_{Z}M\right)c^2 =$$

$$(94 * 1.6726231 + 145 * 1.6749286 - 239.0522 * 1.6605402) * 10^{-27} * 9.10^{16} = 28.22.10^{-11}\ \mathrm{J}$$

Converting to MeV using the fact that MeV = $1.6.10^{-13}$ J gives: E_B = 1763 MeV

b. [40%]

$$\Delta m = mass\left({}^{239}_{94}Pu\right) - mass\left({}^{142}_{56}Ba\right) - mass\left({}^{95}_{38}Sr\right) - 2.mass\left({}^{1}_{0}n\right)$$

Δm = 239.0522-141.9031-94.9144-2*1.008665 = 0.21737 amu.

With E=0.21737x1.6605402x10^{-27} x 9x10^{16}=3.248564609466 x10^{-11} J or 203.04 MeV

We thus get Δm = 203.04 MeV

Or, 1 amu = 1.6605402x10^{-27} x 9x10^{16}/1.6 10^{-13} = 934.0538625 MeV

And 0.21737 amu equals 203.04 MeV

c. [30%]

A nuclear power plant with power of 1 GW(e) and a load factor of 80% yields: 10^9*0.8*365*24*3600 = 2.52288.10^{16} J.

The nuclear power plant requires three times the amount of energy because the efficiency is 33.3%.

The thermal energy demand thus equals: 7.56864$10^{16}$ J.

The energy of one kg plutonium-239 is 8.13.10^{13} J

We therefore need 75.6 10^{15}/8.13 10^{13} = 930.952 kg.

4. Fuel cell

a. [10%]

$$V^0 = -\frac{\Delta G^0}{nF} = -\frac{\Delta H - T\Delta S}{nF} = -\frac{\left(-285800 - 298\times(-163.2)\right)}{2\times 96487} = 1.229 \text{ V}$$

b. [10%] T = 200 °C instead of 25 °C

$$V^0 = -\frac{\Delta G^0}{nF} = -\frac{\Delta H - T\Delta S}{nF} = -\frac{\left(-285800 - 473\times(-163.2)\right)}{2\times 96487} = 1.081 \text{ V}$$

c. [15%]

Assume

$$\boxed{\begin{aligned} V_1 &= V^0 + \frac{RT_1}{nF}\ln\frac{P_{H_2}\sqrt{P_{O_2}}}{P_{H_2O}} = V^0 + \frac{RT_1}{nF}C \\ V_2 &= V^0 + \frac{RT_2}{nF}\ln\frac{P_{H_2}\sqrt{P_{O_2}}}{P_{H_2O}} = V^0 + \frac{RT_2}{nF}C \end{aligned}}$$

Thus $\Delta V_T = V_2 - V_1 = \frac{RT_2}{nF}C - \frac{RT_1}{nF}C = \frac{RC}{nF}\left(T_2 - T_1\right)$

Or

$$\boxed{\begin{aligned} V_1 &= V^0 + \frac{RT_1}{nF}\ln\frac{P_{H_2}\sqrt{P_{O_2}}}{P_{H_2O}} \Rightarrow V_1 - V^0 = \frac{RT_1}{nF}C \\ V_2 &= V^0 + \frac{RT_2}{nF}\ln\frac{P_{H_2}\sqrt{P_{O_2}}}{P_{H_2O}} \Rightarrow V_2 - V^0 = \frac{RT_2}{nF}C \end{aligned}}$$

Thus $\frac{V_2 - V^0}{V_1 - V^0} = \frac{RC}{nF}\frac{T_2}{T_1}$

d. [15%]

Assume

$$\boxed{\begin{aligned} V_1 &= V^0 + \frac{RT}{nF}\ln\frac{P_1\sqrt{P_{O_2}}}{P_{H_2O}} = V^0 + \frac{RT}{nF}\left(\ln P_1 + \ln\frac{\sqrt{P_{O_2}}}{P_{H_2O}}\right) \\ V_2 &= V^0 + \frac{RT}{nF}\ln\frac{P_2\sqrt{P_{O_2}}}{P_{H_2O}} = V^0 + \frac{RT}{nF}\left(\ln P_2 + \ln\frac{\sqrt{P_{O_2}}}{P_{H_2O}}\right) \end{aligned}}$$

Thus $\Delta V_P = V_2 - V_1 = \frac{RT}{nF}\ln P_2 - \frac{RT}{nF}\ln P_1 = \frac{RT}{nF}\ln\frac{P_2}{P_1}$

e. [10%] With $P_{H_2} = P_{O_2} = 5$ bar and $P_{H_2O} = 1$ bar C equals $C = \ln \frac{P_{H_2}\sqrt{P_{O_2}}}{P_{H_2O}} \ln \frac{5\sqrt{5}}{1} = 2.414$

→ $$\Delta V_T = V_2 - V_1 = \frac{RC}{nF}(T_2 - T_1) = \frac{8.3144 \times 2.414}{2 \times 96487}(50) = 0.104 \text{ mV}$$

f. [10%] With T=473 K

→ $$\Delta V_P = V_2 - V_1 = \frac{RT}{nF} \ln \frac{P_2}{P_1} = \frac{8.3144 \times 473}{2 \times 96487} \ln(2) = 14.126 \text{ mV}$$

g. [15%]

Use $\eta = \frac{\Delta G}{\Delta H} \frac{V_{actual}}{V_{ideal}} = \frac{\Delta H - T\Delta S}{\Delta H} \frac{V_{actual}}{V_{ideal}} = \frac{-285800 - (473 \times -163.2)}{-285800} \frac{0.7}{1.081} = 47.3 \%$

Or

$$\eta = \frac{\Delta G}{\Delta H} \frac{V_{actual}}{V_{ideal}} = \frac{\Delta H - T\Delta S}{\Delta H} \frac{V_{actual}}{V^0} = -\frac{\Delta H - T\Delta S}{\Delta H} \frac{nFV_{actual}}{\Delta H - T\Delta S} = -\frac{nFV_{actual}}{\Delta H} = -\frac{2 \times 96487 \times 0.7}{-285800} = 47.3 \%$$

h. [15%]

Total electrical power generated equals $P = V_{cell} \times I_{cell} = 0.7 \times 0.6 \times 100 = 42$ W

Total power generated equals $P = V_{ideal} \times I_{cell} = 1.081 \times 0.6 \times 100 = 64.86$ W

Total heat produced thus is 64.86-42=22.86 W. (35.2%)

CHP use is clear; amount of produced heat is appreciable.