Intermediate Exam

Master Course Energy Conversion and Environmental Technologies
Date: 16 December 2005

This exam comprises four (4) questions. For each question 2.5 points can be earned, for each subquestion a certain percentage of this.

Please read the questions carefully.

Write your name and student number clearly on the top left corner of all answers that you hand in.

Good luck!

1. Efficiency of a gas turbine

Consider a gas turbine that has the following characteristics:
- state 1: compressor inlet, pressure 1 bar, temperature 25 °C
- state 2: compressor outlet, pressure 10 bar, temperature 450 °C
- state 3: turbine inlet, pressure 10 bar, temperature 1000 °C
- state 4: turbine outlet, pressure 1 bar, temperature 525 °C

The thermal efficiency of this air-standard Brayton cycle can be calculated with the usual equation using differences of enthalpies of different states.

a. Draw a flow diagram of the gas turbine, including all components, heat flows and work performed [10%]
b. Draw the process cycle in the attached T-S diagram [10%]
c. Calculate the efficiency of the gas turbine, using the characteristics given above, while neglecting all further losses). [20%]

Suppose that both the compressor and the turbine are isentropic, i.e., they are irreversible in the thermodynamic meaning of the word).

d. Draw this process cycle in the attached T-S diagram [10%]
e. Calculate the efficiency of the gas turbine in this situation. [20%]
f. What are the isentropic efficiencies of the compressor and the turbine? [10%]

g. Would an increase of turbine pressure lead to an increase in the turbine efficiency? Motivate your answer. [10%]

h. What is the disadvantage of increasing the pressure? [10%]

2. Photovoltaic conversion

Consider an ideal p-n junction (diode) solar cell under illumination with a light power density P_{in} (W/m^2). The current-voltage characteristic ("curve") is described by:

$$J(V) = J_0\left[\exp\left(\frac{qV}{kT}\right)-1\right]-J_L = J_0\left[\exp\left(\frac{V}{V_{th}}\right)-1\right]-J_L$$

in which *J(V)* is the current density as a function of voltage over the cell terminals *V*, J_o is the saturation current density and J_L is the light-generated current density (A/m^2, usually mA/cm^2). *q* is the elementary charge (1.6 10^{-19} C), *k* is Boltzmann's constant (1.38 10^{-23} J/K), *T* is the absolute temperature (K). Note that $J_L \gg J_0$.

a. Sketch the relevant part of the *J(V)* characteristic, indicating the points of special interest: short-circuit current density J_{sc}, open-circuit voltage V_{oc}, and maximum power point (V_{mpp}, J_{mpp}). The fill factor $FF \equiv (V_{mpp}\times J_{mpp})/(V_{oc}\times J_{sc})$. Define the efficiency of the solar cell in terms of the parameters given. [20%]

b. Give an expression for V_{oc} as a function of J_L. Use the current-voltage characteristic given above and first determine the current at open-circuit conditions. Note: in all relevant cases $J_L = J_{sc}$ [20%]

c. If the solar cell has an open circuit voltage of 620 mV under 1 sun illumination (1000 watt/m^2), what would the open circuit voltage become under concentrated light of 10, 100 and 1000 suns?
Assume that both temperature and J_0 remain constant and that J_L is linearly dependent on illumination intensity (thus $J_{L,10\ Suns} = J_{L,1\ Sun} \times 10$). [Hint: use the formula outlined above to determine the actual value of V_{th}.. In case you have forgotten some math: ln(*ab*) = ln(*a*) + ln(*b*)] [20%]

d. If the solar cell in [**c**] had an efficiency of 18% at 1 sun, what would be reasonable values of the short-circuit current density and the current density at maximum power? You do not need to be very accurate, i.e., use a fill factor value typical for high efficiency solar cells. [20%]

In practice solar cells show different current-voltage behaviour:

$$J(V) = J_0\left[\exp\left(\frac{qV}{nkT}\right)-1\right]-J_L = J_0\left[\exp\left(\frac{V}{nV_{th}}\right)-1\right]-J_L$$

in which *n* is the diode ideality factor. Normally 1 < *n* < 2. Note that J_o' may be very different from J_o.

e. Assuming that the solar cell has the same short-circuit current density J_{sc} and open-circuit voltage V_{oc}, would the efficiency of such a cell be lower, higher, or the same as the device mentioned in [**c**]? Do not just answer "yes" or "no", but also describe in words why that is. Draw a sketch of the two curves in one figure. [20%]

3. Nuclear fission

Fission of heavy elements generates energy because the binding energy of fission products is larger than the original fission fuel itself. The generated energy is used to heat water to steam, which allows for electricity generation by means of a classic steam cycle. Assume that in a certain nuclear fission reactor plutonium-239 is used as fuel.

Plutonium-239 is a human made element that contains 94 protons and 145 neutrons and is denoted as ${}^{239}_{94}\mathrm{Pu}$. The mass of ${}^{239}_{94}\mathrm{Pu}$ is 239.0522 amu (atomic mass units); its density is 19,74 kg/dm^3; its half-life is 24360 years and the cross section for fission is 740 barn.

A neutron can split plutonium as follows:

$${}^{1}_{0}\mathrm{n} + {}^{239}_{94}\mathrm{Pu} \rightarrow {}^{142}_{56}\mathrm{Ba} + {}^{95}_{38}\mathrm{Sr} + 3\,{}^{1}_{0}\mathrm{n}$$

where the mass of barium-142 is 141.9031 amu and the mass of strontium-95 is 94.9144 amu. Use in the calculations $c = 3\times10^{8}$ m/s, $m_{proton} = 1.6726231\times10^{-27}$ kg, $m_{neutron} = 1.6749286\times10^{-27}$ kg, 1 amu $= 1.6605402\times10^{-27}$ and 1 MeV = 1.6×10^{-13} J.

a. Calculate the total binding energy of plutonium-239 in MeV using Einstein's famous relation $E = mc^2$. [30%]

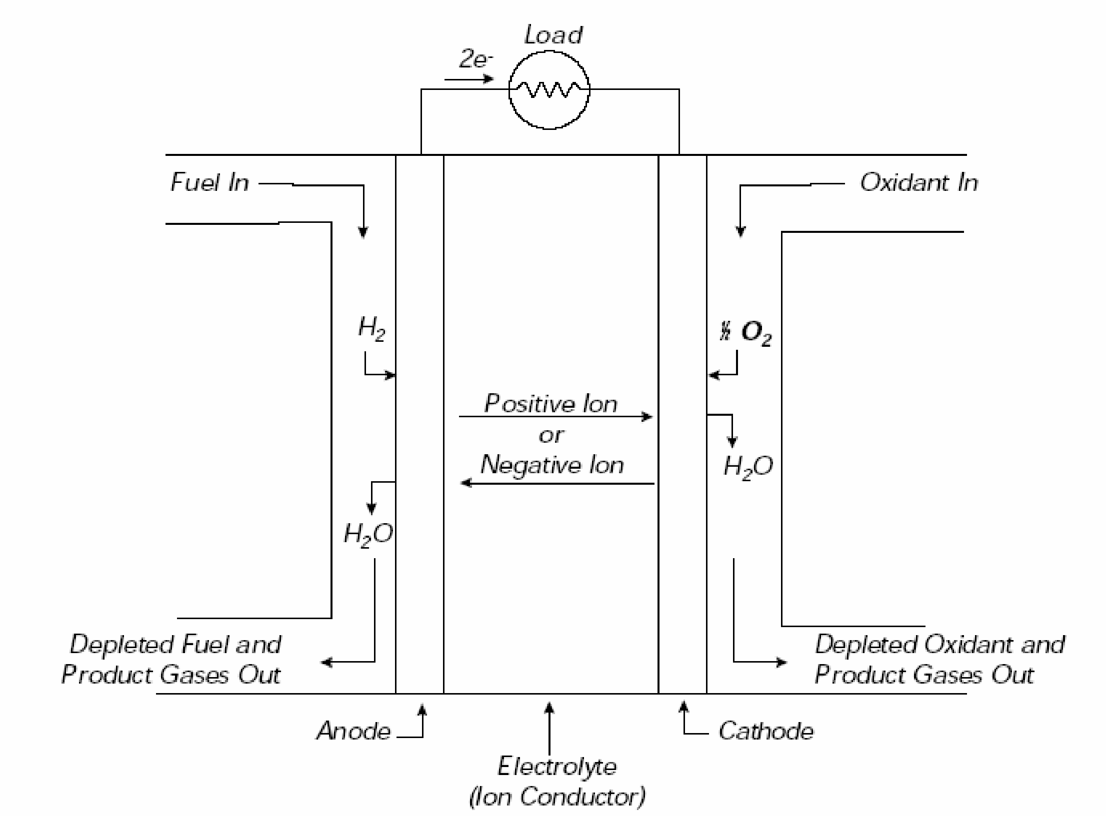
b. Calculate the energy in MeV that is liberated in the fission reaction of plutonium-239. [40%]

Assume that the energy of one kilogram of Plutonium-239 equals 8.13×10^{13} J.

c. Calculate the amount of Plutonium-239 consumed per year in a nuclear power station with a capacity of 1 GW(e) that operates with a load factor of 80%. Note that the electric efficiency of the nuclear power station is 33.3%. [Hint: first calculate the amount of electric energy produced] [30%]

4. Fuel cell

Fuel cells are electrochemical devices that convert the chemical energy of a reaction directly into electrical energy. The basic physical structure or building block of a fuel cell consists of an electrolyte layer in contact with a porous anode and cathode on either side. A schematic representation of a fuel cell with the reactant/product gases and the ion conduction flow directions through the cell is shown in the figure below (from Hirschenhofer *et al.*, Fuel cell handbook, 1998).



The most common classification of fuel cells is by the type of electrolyte used in the cells; here we consider only the phosphoric acid (H_3PO_4) fuel cell (PAFC), which typically operates at a temperature of about 200°C for PAFC. The ideal performance of the PAFC depends on the electrochemical reactions that occur with hydrogen and oxygen. Low-temperature fuel cells, such as the PAFC require noble metal catalysts to achieve practical reaction rates at the anode and cathode, and H_2 is the only acceptable fuel.

Anode reaction: $H_2 \rightarrow 2H^+ + 2e^-$

Cathode reaction: $\frac{1}{2}O_2 + 2H^+ + 2e^- \rightarrow H_2O$

The ideal performance of a fuel cell is defined by its Nernst potential, represented as cell voltage:

$$V = V^0 + \frac{RT}{nF} \ln \frac{P_{H_2}\sqrt{P_{O_2}}}{P_{H_2O}}$$

where R is the universal gas constant (8.3144 J/mol/K), F the Faraday constant (96487 C/mol), *n* the number of electrons participating in the reaction, and *P* the pressures. The Nernst equation provides a relationship between the ideal standard potential (V^0) for the cell reaction and the ideal equilibrium potential (V) at other temperatures and partial pressures of reactants and products. Once the ideal potential at standard conditions is known, the ideal voltage can be determined at other temperatures and pressures through the use of these equations.

The ideal standard potential V^0 can be calculated using the Gibbs free energy change ΔG in the fuel cell process defined as usual as $\Delta G = \Delta H - T\Delta S$. The amount of electrical work obtainable from a fuel cell equals $\mathrm{W} = \Delta G = -nFV$, at standard conditions this is $\mathrm{W}^0 = \Delta G^0 = -nFV^0$.

a. Calculate the ideal standard potential V^0 using enthalpy $\Delta H = -285.8$ kJ and entropy $\Delta S = -163.2$ J/K. [10%]
b. Calculate the ideal standard potential V^0 at an operating temperature of 200°C (as ΔH and ΔS change only slightly in practice, assume them to be constant) [10%]
c. Derive an equation for the change in voltage resulting from a change in temperature, assuming all pressures to be constant, however realize that V^0 is also dependent on temperature. [Hint: compare the Nernst equations for temperatures T_1 and T_2] [15%]
d. Derive an equation for the change in voltage resulting from a change in hydrogen pressure, assuming all other pressures and temperature to be constant. [Hint: compare the Nernst equations for pressure P_1 and P_2] [15%]

Actual conditions for the PAFC are a temperature range of 175°C < T < 225°C and a pressure range of 5 bar < P < 10 bar.

e. Use the equation derived in [**c**] to calculate the change in voltage going from 175°C to 225°C, at $P_{H_2} = P_{O_2} = 5$ bar and $P_{H_2O} = 1$ bar. [10%]
f. Use the equation derived in [**d**] to calculate the change in voltage going from 5 to 10 bar, at a temperature of 200°C. [10%]

The efficiency of an actual fuel cell can be expressed in terms of the ratio of the actual cell voltage to the ideal cell voltage (the actual cell voltage is less than the ideal cell voltage because of various losses):

$$\eta = \frac{\Delta G}{\Delta H}\frac{V_{actual}}{V_{ideal}}$$

g. Calculate the efficiency at an operating temperature of 200°C for a PAFC that has an actual voltage of 0.7 V. [15%]
h. Assume that a 100 cm^2 PAFC operates at 1 bar and 200°C at 0.7 V and generates 600 mA/cm^2. This leads to a considerable amount of heat production next to the generated electricity, which can be calculated to be the difference of total power (based on ideal voltage) and electrical power (based on actual voltage). What is the heat output of the PAFC? Do you now understand why these fuel cells may be used as CHP plants? [15%]